CALCULUS I/MATH 150

SHANNON GRACEY

EXAM 2/PART 1/CHAPTERS 3.1, 3.3-3.6

- π 50 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π NO GRAPHING CALCULATOR IS PERMITTED
- π PROVIDE EXACT ANSWERS (NO DECIMALS PLEASE)

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED. THIS MEANS NO BATHROOM BREAKS…

NAME Key

MATH 150/GRACEY NAME_____

EXAM 2/PART 1/50 POINTS POSSIBLE Part 1_____Part 2_____ E2_____%

NO CALCULATOR

YOU MUST TURN IN THIS PART OF THE EXAM BEFORE YOU BEGIN PART 2. YOU MAY NOT RETURN TO THIS PART OF THE EXAM ONCE YOU BEGIN PART 2.

Completely analyze the following function using the skills you learned in 3.1, and 3.3-3.6 of our text. Be sure to write "NONE" if this function does not have a characteristic listed below. If you leave an item blank, it will be marked wrong.

Consider the function $f(x)=\frac{2x^2-13x+15}{x-3}$ $\rightarrow$ $f(x)=\frac{(2x-3)(x-5)}{x-3}$

1. Give the ordered pairs representing the intercepts.

x-int: $0=(2x-3)(x-5)$

$2x-3=0$ $x-5=0$

$x=3/2$ $x=5$

y-int: $f(0)=\frac{2(0)^2-13(0)+15}{(0)-3}=\frac{15}{-3}=-5$

a. (1 point) *x*-intercept: $(\frac{3}{2},0),(5,0)$

b. (1 point) *y*-intercept: $(0,-5)$

2. Write the lines representing the vertical and horizontal asymptotes.

a. (2 points) Vertical asymptote(s): $x=3$

$x-3=0 \rightarrow x=3$

b. (2 points) Horizontal asymptote(s): NONE

$\lim_{x\to\infty}\frac{(2x^2-13x+15)\frac{1}{x}}{(x-3)\frac{1}{x}}=\lim_{x\to\infty}\frac{2x-13+\frac{15}{x}}{1-\frac{3}{x}}$

$=\infty$

$f(x)=\dfrac{2x^2-13x+15}{x-3}$

130
65
195

3. (8 points) Find the critical numbers for f.

$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{2x^2-13x+15}{x-3}\right)$

$f'(x) = \dfrac{(4x-13)(x-3) - (2x^2-13x+15)(1)}{(x-3)^2}$

$f'(x) = \dfrac{4x^2-25x+39-2x^2+13x-15}{(x-3)^2}$

$f'(x) = \dfrac{2x^2-12x+24}{(x-3)^2}$

$f'(x) = \dfrac{2(x^2-6x+12)}{(x-3)^2}$

$0 = 2(x^2-6x+12)$

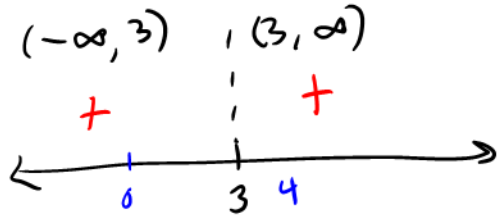
$0 = x^2-6x+12$

$x = \dfrac{6 \pm \sqrt{36-4(1)(12)}}{2}$

$x = \dfrac{6 \pm \sqrt{-12}}{2}$ imaginary

no C.N.

4. (3 points) Run the test for increasing/decreasing intervals.



$f'(0) = \dfrac{2((0)^2-6(0)+12)}{(0-3)^2} = \dfrac{24}{9} > 0$

$f'(4) = \dfrac{2((4)^2-6(4)+12)}{(4)-3)^2} = 8 > 0$

a. (2 points) f is increasing on $(-\infty, 3) \cup (3, \infty)$

b. (2 points) f is decreasing on NONE

c. (2 points) Give the ordered pairs where relative minima occur.

NONE

d. (2 points) Give the ordered pairs where relative maxima occur.

NONE

$$f(x)=\frac{2x^2-13+15}{x-3}$$

5. Test for concavity and find any points of inflection.

   a. (15 points) Run the test for concavity.

$\frac{d}{dx}f'(x)=\frac{d}{dx}\left(\frac{2(x^2-6x+12)}{(x-3)^2}\right)$

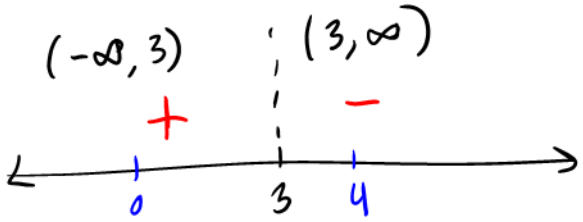
$f''(x)=\frac{2(2x-6)(x-3)^{2\cdot1}-2(x^2-6x+12)[2(x-3)^{1}(1)]}{(x-3)^{4}}$

$f''(x)=\frac{2(2x^2-12x+18)-4x^2+24x-48}{(x-3)^3}$

$f''(x)=\frac{4x^2-24x+36-4x^2+24x-48}{(x-3)^3}$

$f''(x)=\frac{-12}{(x-3)^3}$

no zeros



$f''(0)=\frac{-12}{(0-3)^3}>0$

$f''(4)=\frac{-12}{(4-3)^3}<0$

   b. (2 points) f is concave upwards on $(-\infty, 3)$

   c. (2 points) f is concave downwards on $(3, \infty)$

   d. (2 points) Give the ordered pairs which represent points of inflection.

NONE

$$f(x)=\frac{2x^2-13x+15}{x-3}$$

6. (4 points) Sketch the graph, using the information from your analysis. Find additional ordered pairs as needed. Be sure to label your axes and write in the scale.

$f(10)=\frac{2(100)-130+15}{10-3}=\frac{85}{7}=12\frac{1}{7}$

$f(1)=\frac{2(1)+13+15}{-4}=-\frac{15}{2}$

$f(4)=\frac{2(16)-52+15}{4-3}=-5$